Géométrie et Topologie des Singularités

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Colloque annuel du GDR Singularités

Programme

Cours (4 séances de 55 minutes)

- **Enrique Artal Bartolo** (Université de Saragosse, Espagne)

  Topology of hyperplane arrangements

  The goal of this lectures is to study the relationship between combinatorics and topology of hyperplane arrangements in a complex projective space. We will compare the cohomology ring of the complement with the Orlik-Solomon algebra which is defined in terms of combinatorics. In order to study the fundamental group of the complements, one can restrict to the case of line arrangements and we will explain how to derive from braid monodromy the informations about the fundamental group and the topology of the complement. One important case where a lot of topological information is known is the case of real hyperplanearrangements, where the real picture encode information about the topology of the complexifications. Finally we will explain the examples of Rybnikov and Artal-Carmona-Cogolludo-Marco which show that combinatorics do not determine the topology.

- **Andras Nemethi** (Rényi Mathematical Institute, Budapest, Hongrie)

  Lattice (co)homology of normal surface singularities

  To any abstract topological type of a normal surface singularity (equivalently, to any negative definite plumbed 3-manifold) we associate a graded $\mathbf{Z}[U]$ module. The construction is combinatorial from the plumbing (resoltuion) graph. The homology groups create the bridge between different aspects of the singularity. From topological point of view, it is related with the Heegaard-Floer homology of the link (in particular, to the Seiberg-Witten invarint. of the link). On the other hand, its properties are closely related with the classification of singularities (e.g., it characterizes the rational and elliptic singularities). Moreover, it provides upper bounds for some analytic invariants as well. The 0-homology corresponds to the homology associated with 'graded roots'.

- **Hansjörg Geiges** (Univ. Kln, Allemagne)

  Higher dimensional contact geometry

  1. Basics of Contact Topology
  2. Contact Surgery
  3. Open Books I
  4. Open Books II
  5. Other Constructions of Contact Manifolds

Conférences de 45 minutes

- **Daniel Barlet** (Université de Nancy, IUF)

  Sur les fonctions a singularité de dimension 1

  The aim of the present talk is to construct analytic invariants for a germ of an holomorphic function having a one dimensionnal critical locus S.

  More precisely, aside the Brieskorn (a, b)-module at the origin and a (locally constant along $S^* := S \setminus \{0\}$) sheaf $\hat{H}^n$ of (a, b)-modules associated to the transversal hypersurface singularities along each connected component of S^*, we construct also (a, b)-modules "with supports" E_c and $E_{c \cap S}$. Remark that, even in the case of a germ with an isolated singularity, the "dual" Brieskorn E_c was not noticed until the recent paper [B.05]. In this case $E_{c \cap S}$ coïncides with the usual Brieskorn (a, b)-module E.

An interesting consequence of the local study along S^* is the proof that for a germ with an isolated singularity, the largest sub-(a, b)-module having a simple pole in its Brieskorn-(a, b)-module is independant of the choice of a reduced equation for the corresponding hypersurface germ.

We also give precise relations between these various (a, b)-modules via an exact commutative diagram. This is an (a, b)-linear version of the tangling phenomenon for consecutive strata we have previously studied in the opologicalsetting (see [B.91], [B.02] and [B.06 c)]) for the localized Gauss-Manin system of f. Finally we show that in this situation there exists a non-degenerate (a, b)-sesquilinear pairing

$$h : E \times E_{c\cap S} \to |\Sigma'|^2$$

where $|\Sigma''|^2$ is the space of formal asymptotic expansions at the origin for fiberintegrals. This generalizes the canonical hermitian form defined in [B.85] for the isolated singularity case (for the (a, b)-module version see [B.05]). Its topological analogue (for the eigenvalue 1 of the monodromy) is the non-degenerate sesquilinear pairing

$$h : H^n_{c\cap S}(F, \mathbf{C})_{=1} = 1 \times H^n(F, \mathbf{C})_{=1} \to \mathbf{C}$$

defined in [B.06 c)] for an arbitrary germ such that the eigenvalue 1 of the monodromy acting on the reduced cohomology of the Milnor fibers only appears along a curve. Then we show this sesquilinear pairing is related to the non-degenerate sesquilinear pairing introduced on the sheaf $\hat{H}^n$ via the canonical hermitian form of the transversal hypersurface singularities.

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Key Words : Hypersurface, Non Isolated Singularity, Vanishing Cycles, Tangling of Strata, Brieskorn module, (a,b)-modules.

- **Arnaud Bodin** (Université Lille 1)

Integer points on generic fibers/ Points entiers sur les fibres génériques

Let $P(x, y)$ be a non-composite rational polynomial and $k \in \mathbf{Q}$ be a generic value. If the curve $(P(x, y) = k)$ admits an infinite number of integer points then $P(x, y)$ is algebraically equivalent to the polynomial

x. Moreover for such curves (and others) we give a sharp bound for the number of integer points (x, y) with x and y bounded.

- **Gabor Braun** (Rényi Mathematical Institute, Budapest, Hongrie)

  Topological invariants of special surface singularities

  The talk complements the course "Normal singularities of surfaces" by Andrs Nmethi. I will speak on combinatorical aspects of local surface singularities focusing on two recent results obtained jointly with Andrs: (1) the topology completely determines the equisingular type of Newton non-degenerate surface singularities, (2) the Seiberg?Witten invariant conjecture for splice-quotient singularities.

- **Clément Caubel**

  Structures de contact et singularités non isolées

  J'expliquerai comment associer a certaines singularités de variétés analytiques complexes une variété de contact, ceci généralisant la notion de bord de contact des singularités isolées. Je donnerai ensuite des exemples de singularités non isolées de surfaces dont les varietes de contact associées ne sont le bord de contact d'aucune singularité isolée, meme si elles peuvent l'être a difféomorphisme près."

- **Jose Ignacio Cogolludo**

  Plane curves, pencils, and combinatorial types

  Given a plane algebraic curve $\mathcal{C} \subset \mathbf{CP}^2$, we will define an invariant called the *weak combinatorial type* of $\mathcal{C}$.

  This object determines some interesting topological invariants of the pair $(\mathbf{CP}^2, \mathcal{C})$ such as the cohomology algebra of the complement $S_{\mathcal{C}} := \mathbf{CP}^2 \setminus \mathcal{C}$ or its resonance varieties. This will allow us to show that the pencils contained in a curve (analogue to Noether's fundamental Theorem) are also of a combinatorial nature.

  Using this approach we will solve the problem of formality of $S_{\mathcal{C}}$ and give some insight into the algebra structure of differential logarithmic forms on $S_{\mathcal{C}}$.

- **Delphine Dupont**

TBA

- **Javier Fernandez de Bobadilla**

  On topological equisingularity for surfaces

  We will introduce the notion of Equisingularity at the Normalisation and explain some of its applications to equisingularity of parametrised and embedded surfaces in C3.

- **Mauricio Garay**

  Intégrabilité quantique et anomalies topologiques

  (*travail en commun avec Duco van Straten*)

- **Pedro D. Gonzalez Perez**

  Multi-Harnack smoothings of real plane branches

  (*(joint work with J.-J. Risler*)

  Résumé : A smoothing of a germ $(C, 0)$ of real algebraic plane curve singularity is a real analytic family of real algebraic plane curves C_t, for $t \in [0, 1]$, such that $C_0 = C$ and $C_1 = C'$ and C_t for $0 < t \ll 1$ is non singular and transversal to the boundary of a Milnor ball B of the singularity $(C, 0)$. The real part $\mathbf{R}C'$ of C' in the Milnor ball consists of finitely many ovals and non closed components. A M-real algebraic curve (resp. a M-smoothing) reaches the *Harnack bound* on the number of connected components of the real part (in the Milnor ball). There exists real projective M-curves of any degree but M-smoothings do not always exists. The obstructions to the existence of M-smoothings is not well-understood in general. For real plane branches the existence of M-smoothings was shown by Risler by using the blow-up method, a geometrical construction which adapts classical methods of Harnack.

  We present a new method for the construction of M-smoothings of a real plane branch $(C, 0)$ which are obtained as the result of a *sequence of M-smoothings* of the strict transforms $(\tilde{C}_j, o_j)$ at of C at certain infinitely near points in a embedded resolution of $(C, 0)$ constructed with toric morphisms. These intermediate smoothings are constructed by using Viro method to glue the charts of suitable M-curves in certain real projective toric surfaces, associated with a sequence of triangles Δ_j determined by the equisingularity class of $(C, 0)$. Remark that Viro method cannot be applied directly to this case since branches are usually degenerated with respect to its Newton polygon.

Mikhalkin has proven the unicity of the topological type of of those M curves in real projective toric surfaces, in particular in $\mathbf{R}P^2$, which are embedded in *maximal position* with respect to the toric axes. This result is proved by analyzing their corresponding *amoebas* (the amoeba of a curve C is the image of the points $(x, y) \in (\mathbf{C}^*)^2$ in the curve by the map $(x, y) \mapsto (\log |x|, \log |y|)$). We analyze to which extent this result of Mikhalkin admits a suitable reformulation for smoothings of singular points of real plane curves, particularly for *Harnack smoothings*, those M smoothings which are in maximal position with respect to two coordinates lines through the singular point. We prove that the topological type of *Multi-Harnack smoothings*, those smoothings which arise in a sequence of Harnack smoothings of the strict transforms $(\tilde{C}_j, o_j)$, is determined by classical equisingularity class of $(C, 0)$, or equivalently by, the embedded topological type of $(C, 0) \subset (\mathbf{C}^2, 0)$. In addition, we give an example of two Harnack smoothings of a real plane branch $(C, 0)$ with different topological type hence the same statement is not true for Harnack smoothings in general.

- **Lê Dung Trang**

  Conjecture jacobienne et polynomes rationnels

- **Michael Loenne**

  Groupe fondamental de complémentaires de discriminants

  Nous allons rappeler la notion de monodromie de tresses pour les discriminants de singularités d'une hypersurface. Nous présentons notre résultat sur la monodromie de tresses de singularités de Brieskorn-Pham et nous en déduisons une très belle présentation du groupe fondamental du complémentaire du discriminant. Nous obtenons aussi une présentation pour les groupes fondamentaux du complémentaire dans le cas de l'espace des hypersurfaces lisses d'un degré fixé dans un espace projectif fixe.

- **Francoise Michel**

Topologie des bords des fibres de Milnor des germes de surfaces singularités non-isolées.

- **Alexandre Sine**

  Maximalite des variétés toriques en dimension 4

  Le but de cet exposé sera de montrer qu'une varité torique X de dimension 4 est maximale, c'est-à-dire qu'elle vérifie $sum_{i=0}^{4}(b_i(X(\mathbf{R})) = sum_{i=0}^{4}(b_i(X(\mathbf{C}))$ en se basant sur la méthode utilisée dans l'article "Is every toric variety an M-variety?" (Clint Mc Crory, Frederic Bihan, Matthias Franz, Joost Van Hamel 2006)".

- **Otto Van Koert**

  Contact homology of connected sums

  "In this talk we shall give a brief introduction to contact homology, a theory that provides invariants for contact manifolds. Roughly speaking, it works as follows. On a contact manifold we can choose a distinguished vector field, whose closed orbits can be used to generate the chain complex for contact homology. The differential is then defined by counting holomorphic curves between these closed orbits. It turns out that the homology is an invariant of the contact manifold. By finding restrictions on the behavior of these holomorphic curves, we can show that there exists a long exact sequence for contact homology of connected sums."